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Analytic Solution of A Circular Plate Whose Boundary is Elastic Fixed on Elastic Foundation

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ABSTRACT

This paper focuses on analyzing circular plates having elastic limits on elastic foundations. This problem is used as a mathematical model, a boundary-value problem in partial differential equations. The deflection and internal forces in the circular plate are precisely calculated by solving this model. This analysis provides a deeper understanding of the mechanical behavior of circular plate structures and can be a basis for design improvement and optimization in civil engineering and mechanics practice.

Keywords: circular plate, elastic fixed, elastic

1. Introduction

It is useful to calculate deflections and internal forces of the circular plate whose boundary is elastic fixed on an elastic foundation. The term 《elastic fixed》 means a vertical displacement exists and a rotation angle doesn't exist at the boundary. One of practical objects is a boundary of circular chimney's base. Generally, a base of structure with a circular wall belongs to this.

In McFarland,et. al. 1972, the deflection differential equation of circular plate subjected to lateral loads was derived and four kinds of boundary conditions were introduced [1]. By Jawad, 2004, the general solution of deflection of the circular plate resting on elastic foundation was presented [2]. And a solution with a boundary condition was saved also there. By [3] the general solution for the bending nonhomogeneous circular plates resting on an elastic foundation is obtained under arbitrary axial symmetrical loads and boundary conditions. The solutions for large deformation of nonhomogeneous circular plates resting on an elastic foundation are derived. Finally, the boundary condition treated in this paper is different from all the others [4-6].

Derivation of the differential equation for the deflection of a circular plate subjected to lateral loads, as explained by McFarland et al. Al. 1972 was essential in understanding how circular plates respond to lateral loads. The four boundary conditions introduced enable further research on various scenarios, and understanding these conditions is essential for applications in engineering fields such as construction and machine design [9]. Jawad presented a general solution of circular plate deflection that shows how the plate can respond to loads when placed on an elastic foundation, and the saved solution provides a basis for further research on this topic. Furthermore, research on flexible nonhomogeneous circular plates placed on elastic foundations has yielded general solutions, expanding our understanding of how this type of plate can respond to loads, especially under axially symmetric loads and varying boundary conditions [10]. Solutions for large deformations of nonhomogeneous circular plates on elastic foundations have also been found, indicating that the research has taken extreme scenarios into account, which can be crucial in practical applications. The boundary conditions discussed in this paper differ from others, adding to our understanding of how different boundary conditions can influence plate behavior. Overall, this theory statement shows that in-depth research has been conducted on the deflection of circular plates, especially those placed on elastic foundations and those that exhibit nonhomogeneous properties, which is essential for fields such as structural design and structural failure analysis [11] [12].

2. Methods

The previous schema of circular plate whose boundary is transversally loaded on elastic foundation is shown in Fig 1.

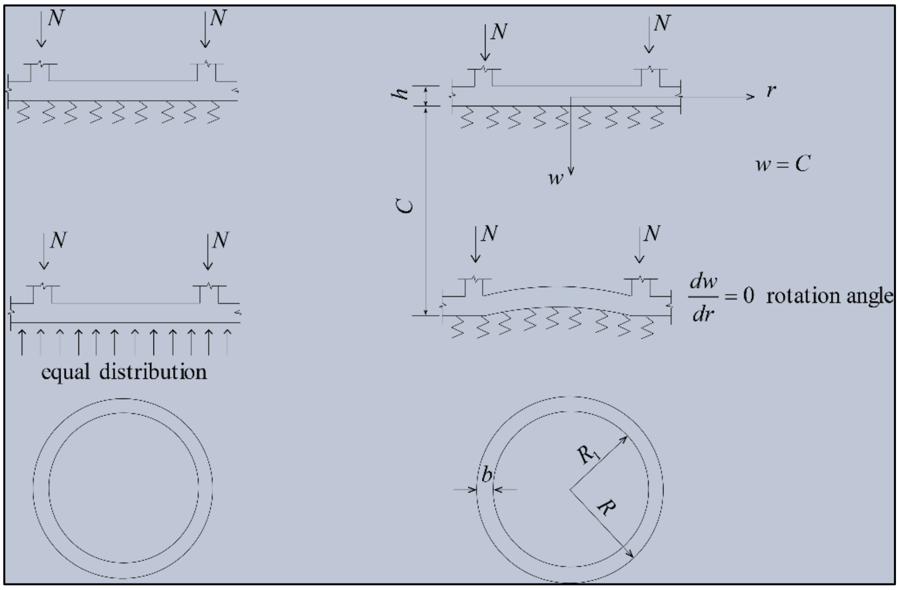

Figure 1 Loads down from the wall distribute to the foundation uniformly

As figure 1 shows, loads down from the wall distribute to the foundation uniformly. Namely we consider the circular plate sustains a load from Winckler’s ground as if a rigid body does. But it is erroneous because the circular plate is rather bent than not. In figure 1 a new schema we study now is introduced. This reflects an actuality more correctly than the previous.

Here

N; a load magnitude per unit length on the circular wall

h; a height of circular plate

R; an outer radius of circular plate

R1; an inner radius of circular plate

b; a thickness of circular wall

w; a deflection of every point on the circular plate

c; a deflection on the boundary of circular plate

k; a foundation modular

As we know, a deflection differential equation of circular plate is

$$\nabla^2\nabla^2 w = \frac{q}{D}$$

Here $q = -kw$, therefore $\nabla^2\nabla^2 w = -\frac{k}{D} w$ (1)

Boundary condition $\left.\begin{matrix} w = C \\ \frac{\partial w}{\partial r} = 0 \end{matrix}\right\}$ when $r = R_1 = R - b$ (2)

Mathematically ① and ② is a boundary value problem of differential equation.

Let's begin solving on this problem.

3. Result and Discussion

In equations (1), (2) let's put $w = W + C$

Then formula (1) becomes $\nabla^2\nabla^2 W + \frac{k}{D} W = -\frac{k}{D} C$ (3)

Formula (2) becomes $\left.\begin{matrix} W = 0 \\ \frac{\partial W}{\partial r} = 0 \end{matrix}\right\}$ when $r = R_1 = R - b$ (4)

In formula (3) W is a function of r ,therefore formula (3) becomes

$$\nabla^2\nabla^2 W(r) + \frac{k}{D} W(r) = -\frac{k}{D} C \qquad (5)$$

For solving this we perform a variable transformation

$$Z = \eta r (\eta = \sqrt[4]{\frac{k}{D}})$$

Then formula (3) becomes $\nabla^2\nabla^2 W(\mathrm{z}) + W(\mathrm{z}) = -C$ and formula (4) becomes

$$\left.\begin{matrix} W(\eta r)\big|_{r=R_1} = 0 \\ W''(\eta r)\big|_{r=R_1} = 0 \end{matrix}\right\} \qquad (6)$$

Next let's consider the equilibrium condition of the plate. A load from the circular wall is equals to the reaction of ground.

$$kC(\pi R^2 - \pi R_1^{\,2}) + \int_0^{R_1} k\left[W(\eta r) + C\right] 2\pi r dr = 2\pi(R - \frac{b}{2})N$$

From $R_1 = R - b$, $2k\pi(R - \frac{b}{2})bC + \int_0^{R-b} k\left[W(\eta r) + C\right] 2\pi r dr = 2\pi(R - \frac{b}{2})N$

$$\pi R^2 Ck + \int_0^{R-b} kW(\eta r) 2\pi r dr = 2\pi(R - \frac{b}{2})N \tag{7}$$

Above equation is used to find out *C*.

A corresponding homogeneous equation with (5) is

$$\nabla^2\nabla^2 W + W = 0 \tag{8}$$

If we consider particular solution of the Eq. $\nabla^2\nabla^2 W(\mathrm{z}) + W(\mathrm{z}) = -C$ to be $W_{s,}$ then W_s=-C.

W_s ; special solution of the Eq. $\nabla^2\nabla^2 W(\mathrm{z}) + W(\mathrm{z}) = -C$

From $\nabla^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}$,

formula ⑧ becomes

$(\nabla^2 + i)(\nabla^2 W - iW) = 0$. (commutativity, linearity).

$(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + i)(\frac{d^2 W}{dr^2} + \frac{1}{r}\frac{dW}{dr} - iW) = 0$ (Here r is denoted by Z).

$$\therefore (\nabla^2 + i)W = 0,\ (\nabla^2 - i)W = 0\ .$$

Generally in an axial symmetric problem, $(\nabla^2 + \mathrm{A})\mathrm{W} = 0$ is zero degree Bessel equation.

Namely, $W'' + \frac{1}{\mathrm{Z}}W' + AW = 0$.

Two homogeneous solutions of this Eq. are

$J_0(\sqrt{A}Z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!n!}(\frac{\sqrt{A}Z}{2})^{2n}$: zero degree Bessel function.

$N_0(\sqrt{A}Z) = \frac{2}{\pi} J_0(\sqrt{A}Z) ln\sqrt{A}Z - \frac{2}{\pi}\left[\sum_{n=0}^{\infty} \frac{(-1)^m}{m!m!}(\frac{\sqrt{A}Z}{2})^{2m} \sum_{k=1}^{m} \frac{1}{k}\right]$: zero degree von Neumann function.

From now on, writing, $\sum_{\mathrm{m}=0}^{\infty} \left[\frac{(-1)^m}{m!m!}(\frac{\sqrt{A}Z}{2})^{2m} \sum_{k=1}^{m} \frac{1}{k}\right] = \Phi(\sqrt{A}Z)$, then

$$N_0(\sqrt{A}Z) = \frac{2}{\pi} J_0(\sqrt{A}Z) ln\sqrt{A}Z - \frac{2}{\pi}\Phi(\sqrt{A}Z) .$$

These two functions are each other independent homogeneous solutions of a zero degree equation.

Let's find out a homogeneous solution when $A = i$.

$$J_0(\sqrt{i}Z) = \sum_{n=0}^{\infty} \frac{(-1)^n i^n}{n!n!}(\frac{Z}{2})^{2n} = a(Z) + ib(Z)$$

Here $a(Z) = \sum_{n=0}^{\infty} \frac{(-1)^{2n} i^{2n}}{(2n)!(2n)!}(\frac{Z}{2})^{4n} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!(2n)!}(\frac{Z}{2})^{4n}$

$$ib(Z) = \sum_{n=0}^{\infty} \frac{(-1)^{2n+1} i^{2n+1}}{(2n+1)!(2n+1)!}(\frac{Z}{2})^{4n+2} = \text{i}\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!(2n+1)!}(\frac{Z}{2})^{4n+2}$$

$$\Phi(\sqrt{i}Z) = \text{Re}\,\Phi(Z) + i\,\text{Im}\,\Phi(Z)$$

$$\text{Re}\,\Phi(Z) = \sum_{n=0}^{\infty}\left[\frac{(-1)^n}{(2n)!(2n)!}(\frac{Z}{2})^{4n} \sum_{k=1}^{2n}\frac{1}{k} \right]$$

$$i\,\text{Im}\,\Phi(Z) = i\sum_{n=0}^{\infty}\left[\frac{(-1)^{n+1}}{(2n+1)!(2n+1)!}(\frac{Z}{2})^{4n+2} \sum_{k=1}^{2n+1}\frac{1}{k} \right]$$

On the other hand, because, $ln(\sqrt{i}Z) = \frac{\pi}{4} i + \ln Z$,

Therefore $\text{N}_0(\sqrt{i}Z) = \text{Re}_\text{N}(\text{Z}) + i\,\text{Im}_N(Z)$

Here $\text{Re}_\text{N}(Z) = \frac{2}{\pi}(a \ln Z - b\frac{\pi}{4} - \text{Re}\,\Phi(Z))$

$$\text{Im}_N(Z) = \frac{2}{\pi}(b \ln Z + \frac{\pi}{4} a - \text{Im}\,\Phi(Z))$$

$$\text{Re}_\text{N}(Z) = \frac{2}{\pi}(a \ln Z - \text{Re}\,\Phi(Z))$$

$$\text{Im}_N(Z) = \frac{2}{\pi}(b \ln Z + b - \text{Im}\,\Phi(Z)) .$$

Finally

$$J_0(\sqrt{i}Z) = a(Z) + ib(Z) ,$$

$$N_0(\sqrt{i}Z)=\mathrm{Re}_N(Z)+i\,\mathrm{Im}_N(Z)$$

Become respectively solution of $(\nabla^2+i)W=0$.

Similarly, about $(\nabla^2-i)W=0$,

$$J_0(\sqrt{-i}Z)=a(Z)-ib(Z)$$

$$N_0(\sqrt{-i}Z)=\mathrm{Re}_N(Z)-i\,\mathrm{Im}_N(Z)$$

Therefore a solution of $(\nabla^2\nabla^2+1)W=(\nabla^2+i)(\nabla^2-i)W=0$ can be expressed by a real number coefficient polynomial. Here four independent homogeneous solutions are $a(Z),\ b(Z),\ \mathrm{Re}_N(Z),\ \mathrm{Im}_N(Z)$.

$$\therefore W=C_1a(Z)+C_2b_{(Z)}+C_3\mathrm{Re}_N\left(Z\right)+C_4\,\mathrm{Im}_N\left(Z\right)-C$$

Now let's decide integral constants $C_1,\ C_2,\ C_3,\ C_4$. Precisely, $W_{(r)}\big|_{r=0}<\infty$.

When $Z\to 0$, then, $a(Z)\to 1,\ b(Z)\to 0,\ \ln(Z)\to -\infty,\ \mathrm{Re}\Phi(Z)\to 0,\ \mathrm{Im}\Phi(Z)\to 0$. $\mathrm{Re}_N(Z)\to -\infty,\ \mathrm{Im}_N(Z)\to 0$. Therefore $C_3=0$.

And this problem is axis symmetric, therefore $W'_{(r)}\big|_{r=0}=0$, $Q_r\big|_{r=0}=0$.

By the elastic theory

$$Q_r=-D\frac{d}{dz}(\nabla^2W)\bigg|_{r=0}=-D\left[\frac{d^3W}{dZ^3}+\frac{d}{dZ}\left(\frac{1}{Z}\frac{dW}{dZ}\right)\right].$$

When $Z\to 0$,

$$\frac{d}{dz}(\nabla^2a(Z))\to 0,\ \ \frac{d}{dz}(\nabla^2b(Z))\to 0$$

$$\frac{d}{dz}(\nabla^2\,\mathrm{Re}\Phi(Z))\to 0,\ \ \frac{d}{dz}(\nabla^2\,\mathrm{Im}\Phi(Z))\to 0$$

We must notice the first term $Z^2\ln Z$ of $I_{mN}(Z)$. The others $\to 0$

$$(Z^2\ln Z)'=2Z\ln Z+Z^2\frac{1}{Z}=2Z\ln Z+Z$$

$$(Z^2\ln Z)''=2\ln Z+2Z\frac{1}{Z}+1=2Z\ln Z+3$$

$$\left[\frac{1}{Z}(Z^2\ln Z)'\right]'=\frac{2}{Z}+0=\frac{2}{Z}$$

$$(Z^2 \ln Z)''' = \frac{2}{Z}$$

$$\therefore \frac{d}{dz}\left[\nabla^2 (Z^2 \ln Z)\right] \to \infty$$

$\therefore$ $I_{mN}(Z) \to \infty$, therefore $C_4=0$.

Hence

$$W = C_1 a(Z) + C_2 b(Z) - C$$

$a(Z)$, $b(Z)$ are series, respectively.

Upon $a(Z)$, $b(Z)$ we take the first, second terms respectively and then derivate, then

$$a(Z) = 1 - \frac{Z^4}{64}$$

$$b(Z) = -\frac{Z^2}{4} + \frac{Z^6}{2304}$$

$$a'(Z) = -\frac{Z^3}{16}$$

$$b'(Z) = -\frac{Z}{2} + \frac{Z^5}{384}$$

$Z = \eta r$, then $W = C_1 a(\eta r) + C_2 b(\eta r) - C$

$W|_{r=R_1} = 0$, therefore $C_1 a(\eta R_1) + C_2 b(\eta R_1) = C$ (9)

$W'|_{r=R_1} = 0$, therefore

$$\eta\left[C_1 a'(\eta R_1) + C_2 b'(\eta R_1)\right] = 0 \quad (10)$$

The equilibrium condition is

$$\pi k C(R^2 - R_1^2) + 2\pi k \int_0^{R_1} \left[C_1 a(\eta r) + C_2 b(\eta r)\right] r dr = P \quad (11)$$

Here $P = 2\pi(R - \frac{b}{2})N = \pi d N$. $d = 2(R - \frac{b}{2})$.

From ⑨, ⑪ , then

$$\left[C_1 a(\eta r)+C_2 b(\eta r)\right](R^2-R_1^2)+2C_1\int_0^{R_1} a(\eta r)rdr+2C_2\int_0^{R_1} b(\eta r)rdr=\frac{P}{\pi k}=\frac{dN}{k} \quad (12)$$

From ⑩, ⑫, then

$$\begin{bmatrix} a'(\eta R_1) & b'(\eta R_1) \\ a(\eta R_1)(R^2-R_1^2)+2\int_0^{R_1} a(\eta r)rdr & b(\eta R_1)(R^2-R_1^2)+2\int_0^{R_1} b(\eta r)rdr \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}=\begin{bmatrix} 0 \\ \frac{P}{\pi k} \end{bmatrix} \quad (13)$$

Let's describe the above formula simply by **A×C=b**.

Here **C**= $(C_1 , C_2)^T$, **A** is a coefficient matrix of **C**, **b**= $(0 , \frac{P}{\pi k})^T$.

And $\mathrm{detA}=|\mathrm{A}|=$

$$=a'(\eta R_1)\left[b(\eta R_1)(R^2-R_1^2)+2\int_0^{R_1} b(\eta r)rdr\right]-b'(\eta R_1)\left[a(\eta R_1)(R^2-R_1^2)+2\int_0^{R_1} a(\eta r)rdr\right] \quad (14)$$

$$\therefore\ C_1=-\frac{P}{\pi k}b'(\eta R_1)\cdot\frac{1}{\mathrm{detA}},\ C_2=-\frac{P}{\pi k}a'(\eta R_1)\cdot\frac{1}{\mathrm{detA}} \quad (15)$$

$$C=\frac{P}{\pi k}\cdot\frac{1}{\mathrm{detA}}\left[-b'(\eta R_1)a(\eta r)+a'(\eta R_1)b(\eta r)\right] \quad (16)$$

$$\therefore W=\frac{P}{\pi k}\cdot\frac{1}{\mathrm{detA}}\left[-b'(\eta R_1)a(\eta r)+a'(\eta R_1)b(\eta r)\right]-C \quad (17)$$

Here

$$a'(\eta R_1)=\left.\frac{da(z)}{dz}\right|_{Z=\eta R_1}$$

$$b'(\eta R_1)=\left.\frac{db(z)}{dz}\right|_{Z=\eta R_1}$$

And $W=C_1 a(Z)+C_2 b(Z)-C$. $w=W+C$.

Therefore $w=C_1 a(\eta r)+C_2 b(\eta r)$.

From $C_1=-\frac{P}{\pi k}b'(\eta R_1)\cdot\frac{1}{\mathrm{detA}}$ and $C_2=-\frac{P}{\pi k}a'(\eta R_1)\cdot\frac{1}{\mathrm{detA}}$,

Then $w=\frac{P}{\pi k}\cdot\frac{1}{\mathrm{detA}}\left[-b'(\eta R_1)a(\eta r)+a'(\eta R_1)b(\eta r)\right]$

In practical computations we take two terms, i.e. the first, second terms of $a(Z),\ b(Z)$.

Till now the deflection of circular plate whose boundary is elastic fixed on Winckler has been studied. In the result the deflection w is described by the polynomial. Till now the deflection of circular plate whose boundary is elastic fixed on Winckler has been studied. In the result the deflection w is described by the polynomial [7]. Circular plate deflection studies how circular plates bend/bend due to the applied load. The edge of the plate is assumed to be held elastically, meaning that the edge of the plate can still move slightly when bending occurs but is still held within the material's elastic limit. This differs from free or locked boundaries, which allow only accessible or zero movement. Winkler developed this theory to describe the deflection pattern of circular plates with elastic edge boundaries. The results show that a polynomial equation can describe the deflection w [8].

Using formulas of the elastic theory, one can calculate internal forces and stresses after saving the deflection w.

$$M_r = -D\left(\frac{d^2w}{dr^2} + \mu\frac{1}{r}\frac{dw}{dr}\right)$$

$$M_\theta = -D\left(\mu\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr}\right)$$

$$Q_r = -D\frac{d}{dr}(\nabla^2 w)$$

$$\sigma_r = \frac{-EZ}{1-\mu^2}\left[\frac{\partial^2 w}{\partial r^2} + \mu\left(\frac{\partial w}{r\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right)\right]$$

$$\sigma_\theta = \frac{-EZ}{1-\mu^2}\left[\mu\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right]$$

$$\tau_{r\theta} = \frac{-EZ}{1-\mu^2}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial\theta}\right).$$

Substituting the deflection $w = C_1 a(\eta r) + C_2 b(\eta r)$ expressed by the polynomial for the above formulas, we can write as follows.

$$M_r = D\eta^2\left[C_1\left(\frac{1-\mu}{\eta r}a'(\eta r) + b(\eta r)\right) + C_2\left(\frac{1-\mu}{\eta r}b'(\eta r) - a(\eta r)\right)\right]$$

$$M_\theta = -D\eta^2\left[C_1\left(\frac{1-\mu}{\eta r}a'(\eta r) - \mu b(\eta r)\right) + C_2\left(\frac{1-\mu}{\eta r}b'(\eta r) + \mu a(\eta r)\right)\right]$$

$$Q_r = -D\eta^3\left[-C_1 b'(\eta r) + C_2 a'(\eta r)\right]$$

$$\sigma_r = \frac{\eta^2 EZ}{1-\mu^2}\left[C_1\left(\frac{1-\mu}{\eta r}a' + b\right) + C_2\left(\frac{1-\mu}{\eta r}b' - a\right)\right]$$

$$\sigma_\theta = \frac{-\eta^2 EZ}{1-\mu^2}\left[C_1\left(\frac{1-\mu}{\eta r}a' - b\right) + C_2\left(\frac{1-\mu}{\eta r}b' + a\right)\right]$$

$$\tau_{r\theta} = 0$$

In this way the deflection and internal forces can be known easily, and then, one can check an accuracy of the method by the applied programs.

Example

Let's consider the process derived in section 2 as an example.

Data:

$R = 3.5m,\ h = 0.3m,\ b = 0.2m,\ k = 2 \cdot 10^4 KN/m^3,\ N$=80KN/m, $E = 2.6 \cdot 10^7 KN/m^2,\ \mu = 0.2$ Let's divide the whole interval ($r = R_1 = R - b$) into ten, and calculate deflection w, internal forces M_r and M_θ analysis by this paper and ANSYS (FEM).

Comparison between these results is

Table 1 presents quantitative data on deflections, apparently measured over several intervals, perhaps in the context of engineering structures or materials. This table has five columns, each representing "Interval," "r(m)," "Deflection (mm) FEM (ANSYS)," "Deflection (mm) This Paper," and "Relative Error (%)." "Interval" appears to refer to different stages of a measurement or experiment, with the table covering 11 intervals. "r(m)" is a parameter that changes over the interval, with values ranging from 0 to 3.3. There are two sets of deflection measurements, "Deflection (mm) FEM (ANSYS)" and "Deflection (mm) This Paper," which appear to be measured in millimeters (mm), and each of these data sets has slightly different values for each interval. The deflection, according to FEM (ANSYS), changed from 1.40765 mm in the first interval to -1.74329 mm in the last interval. In comparison, the deflection, according to this study, changed from 1.418432 mm in the first interval to -1.754 mm in the last interval. "Relative Error (%)" compares the relative error between two sets of deflection measurements, calculated as a percentage ranging from 0.55% to 0.97%. Overall, this table appears to compare two deflection measurement methods - one using FEM (ANSYS) and the other according to this study, as well as providing a measure of the relative error between these two methods.

Table 2 presents the results of two methods of calculating the moment, or a measure of the rotation produced by a force. One method is simulation using ANSYS's FEM (Finite Element Method) software, and the other method is used in the research referenced by the document (called "This paper" in the table). This moment is measured in kilonewton meters (KN·m). The value in each row shows the result of measuring or calculating the moment at a specific point or interval marked by the value 'r(m).' The value 'r(m)' refers to the radial distance in meters from a central point, perhaps in the context of a spherical or cylindrical structure, or it may refer to the distance from a particular reference point. For each interval, three values are given: FEM(ANSYS): This is the moment value generated by the simulation using the ANSYS FEM software. This paper is the moment value produced by the calculation or research method used in this document. Relative error (%): The relative error between the two methods is measured as a percentage. This is calculated based on the difference between the value produced by the method in this study and the value produced by the ANSYS FEM simulation. These figures generally indicate little difference between the two methods, with relatively small relative errors, typically less than 1%. This may indicate that the method used in this study provides very similar results to ANSYS FEM simulations and, therefore, may be considered valid or accurate in the context of this study.

Table 3 compares the results of the FEM (Finite Element Method) model produced by ANSYS with the results proposed in this paper. Both results are measured in Mθ (KN·m) and are also accompanied by relative errors in percentages. At Interval 1, where r is 0, the FEM result is - 28.8065, and the result of this paper is -29.0615, with a relative error of 0.88%. Furthermore, at Interval 2, where r is 0.33, the FEM result is -28.5679, the result of this paper is -28.7516, with a relative error of 0.64%. At Interval 3, where r is 0.66, the FEM result is -27.658, the result of this paper is -27.81, with a relative error of 0.55%. At Interval 4, where r is 0.99, the FEM result is - 25.976, the result of this paper is -26.2015, with a relative error of 0.86%. At Interval 5, where r is 1.32, the FEM result is -23.765, the result of this paper is -23.867, with a relative error of 0.43%. At Interval 6, where r is 1.65, the FEM result is -20.576, the result of this paper is - 20.724, with a relative error of 0.72%. At Interval 7, where r is 1.98, the FEM result is -16.546, the result of this paper is -16.6673, with a relative error of 0.73%. At Interval 8, where r is 2.31, the FEM result is -11.495, the result of this paper is -11.566, with a relative error of 0.61%. At Interval 9, where r is 2.64, the FEM result is -5.2224, and the result of this paper is -5.2678, with a relative error of 0.86%. At Interval 10, where r is 2.97, the FEM result is 2.3978, the result of this paper is 2.4044, with a relative error of 0.28%. Finally, at Interval 11, where r is 3.3, the FEM result is 11.588, the result of this paper is 11.651, with a relative error of 0.54%. From this table, we can see that the results produced by the FEM model are very close to the relative error, which is generally less than 1%.

Table 1. Deflection w

Interval	r(m)	Deflection (mm)		
		FEM(ANSYS)	This paper	Relative error (%)
1	0	1.40765	1.418432	0.76
2	0.33	1.38754	1.396706	0.66
3	0.66	1.32276	1.330484	0.58
4	0.99	1.20478	1.216581	0.97
5	1.32	1.04054	1.049515	0.86
6	1.65	0.81567	0.821241	0.68
7	1.98	0.51597	0.520785	0.92
8	2.31	0.132756	0.133767	0.76
9	2.64	-0.35489	-0.358174	0.92
10	2.97	-0.9727	-0.978	0.55
11	3.3	-1.74329	-1.754	0.63

Table 2. M_r

Interval	r(m)	M_r (KN·m)		
		FEM(ANSYS)	This paper	Relative error (%)
1	0	-28.8065	-29.0615	0.88
2	0.33	-28.298	-28.441	0.503
3	0.66	-26.346	-26.549	0.76
4	0.99	-23.078	-23.294	0.93
5	1.32	-18.394	-18.522	0.69
6	1.65	-11.976	-12.019	0.36
7	1.98	-3.494	-3.51	0.44
8	2.31	7.3219	7.3437	0.29
9	2.64	20.853	20.939	0.42
10	2.97	37.657	37.7357	0.21
11	3.3	58.076	58.2548	0.31

Table 3. M_θ

Interval	r(m)	M_θ (KN·m)		
		FEM(ANSYS)	This paper	Relative error (%)
1	0	-28.8065	-29.0615	0.88
2	0.33	-28.5679	-28.7516	0.64
3	0.66	-27.658	-27.81	0.55
4	0.99	-25.976	-26.2015	0.86
5	1.32	-23.765	-23.867	0.43
6	1.65	-20.576	-20.724	0.72
7	1.98	-16.546	-16.6673	0.73
8	2.31	-11.495	-11.566	0.61
9	2.64	-5.2224	-5.2678	0.86
10	2.97	2.3978	2.4044	0.28
11	3.3	11.588	11.651	0.54

5. Conclusion

Above, we have calculated analytically the magnitude of the deflection and internal force on a circular plate whose edges are elastically supported on a base with elastic properties. This analytical calculation allows us to calculate the response of the plate correctly and efficiently due to the applied load. The elastic model used for the base and edge of the plate takes into account the flexible properties of the material so that it can be closer to actual conditions. Thus, the deflection and internal force calculations obtained from this analysis can be used as a reference for predicting the response of circular plates under similar conditions in the field. Of course, the assumptions and limitations in the developed mathematical model must be considered.

References

[1] McFarland, D. E., Smith, B. L., and Bernhart, W. D., Analysis of plates, New York, NY: Macmillan 1972.

[2] Jawad M. H., Design of Plate and Shell Structures, ASME Press, 2004, 93-97

[3] Ji Zhen-yi, The General Solution for Axial Symmetrical Bending of Nonhomogeneous Circular Plates Resting on an Elastic Foundation, Applied Mathematics and Mechanics, Vol.12, No.9, Sep, 1991, 871-879

[4] Chen Chan- lin and Zhang Li-Ying, The Large Deflection on Problem of Circular Plate on Elastic Foundation and in Conjunction with Linear Elastic Structure, Applied Mathematics and Mechanics, English Edition, Vol.7, No.4, Apr.1986, 341-354

[5] Wang Jiaxin and Liu Jie, Large Deflection of Problem of thin Orthotropic Circular Plate on Elastic Foundation Variable Thickness under Uniform Pressure, Applied Mathematics and Mechanics, English Edition, Vol.17, No.5, May 1996, 477-485

[6] Ji Zhen-yi, Yen Kai-yuan, General Solution for Large Deflection Problems of Nonhomogeneous Circular Plates on Elastic Foundation, Applied Mathematics and Mechanics, English Edition, Vol.13, No.11, Nov, 1993, 995-1007.

[7] Aliha, M. R. M., Bahmani, A., & Akhondi, S. Determination of mode III fracture toughness for different materials using a new designed test configuration. Materials & Design, 86, 863-871. 2015.

[8] Hassan, A. H. A., & Kurgan, N. Bending analysis of thin FGM skew plate resting on Winkler elastic foundation using multi-term extended Kantorovich method. Engineering Science and Technology, an International Journal, 23(4), 788-800. 2020.

[9] Katebi, A., & Selvadurai, A. P. S. A frictionless contact problem for a flexible circular plate and an incompressible non-homogeneous elastic halfspace. International Journal of Mechanical Sciences, 90, 239-245. 2015.

[10] Shariyat, M., & Alipour, M. M. Differential transform vibration and modal stress analyses of circular plates made of two-directional functionally graded materials resting on elastic foundations. Archive of Applied Mechanics, 81, 1289-1306. 2011

[11] Hansson, E. F. Analysis of structural failures in timber structures: Typical causes for failure and failure modes. Engineering Structures, 33(11), 2978-2982. 2011.

[12] Vicario, A. A., & Toland, R. H. Failure criteria and failure analysis of composite structural components. Composite materials, 7, 51. 2016.