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VRPTW Distribution Route Determination with Rigid and Flexible Time Window and Assignment Based on Number of Demand

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ABSTRACT

Vehicle routing problems (VRP) are common in many companies and play an important role in distribution and logistics. Effective routing can significantly reduce expenses and increase customer satisfaction. The research entitled "Determining VRPTW Distribution Routes with Rigid and Flexible Time Windows and Assigning Based on the Demand Numbers" will solve the modified vehicle routing problem with time window (VRPTW) problem by considering constraints on vehicle capacity, delay times, penalty fees for delays, rigid and flexible time windows, and demand numbers. There is one origin depot for the company that will serve their 15 customers. The method used to overcome the problem is mixed-integer linear programming (MILP). In general, this research classifies time windows into three groups, namely rigid, flexible, and rigid and flexible. The time window model chosen is a rigid time window with a cost of IDR 561,965 and no customers are served late. The rigid time window model was chosen because no customers are served late and the cost difference is not significant or can be said to be very small compared to other time window models.
Keyword: Transportation, Route, VRPTW, Rigid and Flexible TW, MILP

ABSTRAK

Masalah vehicle routing problem (VRP) lazim terjadi di banyak perusahaan dan memainkan peran penting dalam distribusi dan logistik. Rute yang efektif dapat secara signifikan mengurangi pengeluaran dan meningkatkan kepuasan pelanggan. Penelitian yang berjudul "Penentuan Rute Distribusi VRPTW dengan Jendela Waktu Kaku dan Fleksibel serta Penugasan Berdasarkan Jumlah Permintaan" ini akan mencari solusi untuk kasus VRPTW yang dimodifikasi dengan mempertimbangkan constraints jenis kendaraan dan kapasitasnya, waktu keterlambatan, biaya penalti untuk keterlambatan, time window kaku dan fleksibel, jarak, dan jumlah permintaan. Terdapat satu depot asal untuk perusahaan yang akan melayani 15 pelanggannya. Metode yang digunakan untuk menyelesaikan permasalahan adalah mixed-integer linear programming (MILP). Secara umum, penelitian ini mengklasifikasikan time window ke dalam tiga kelompok, yaitu kaku, fleksibel, dan kaku dan fleksibel. Model time window yang terpilih adalah time window kaku dengan biaya Rp561.965. Model time window kaku terpilih karena tidak ada pelanggan yang terlambat dilayani serta selisih biaya yang tidak signifikan atau bisa dikatakan sangat kecil dibandingkan model time window lainnya.
Keyword: Transportasi, Rute, VRPTW, Kaku dan Fleksibel, MILP

1. Introduction

Vehicle routing problem (VRP) aims to identify the optimal route to deliver goods from distribution centers to various destinations by minimizing costs [1]. This problem is prevalent in many companies and plays a crucial role in distribution and logistics. Effective routes can significantly reduce expenses and increase customer satisfaction. Transportation plays a crucial role in the effectiveness of the process in distribution. In VRP, the goal is to find a route, starting and ending in the single place, to visit some destinations. The capacity of both depots and vehicles affects the design of routes that meet all customer demands.

Several complex variations of the classic VRP have been developed, including vehicle routing problem with time window (VRPTW) [2]. VRPTW is a complex combinatorial optimization challenge in the field of logistics and distribution, where the objective is to find a series of optimal routes for a fleet of vehicles to serve a set of customers within a given time frame [3]. Each vehicle starts from the central depot, delivers goods to a subset of customers, and returns to the depot, while adhering to restrictions such as vehicle capacity and customer time windows [4]. This problem extends the classic VRP by including a time constraint, making it more difficult to solve. Various exact and heuristic methods, such as branch-and-bound, genetic algorithms, and simulated annealing, have been developed to address VRPTW due to its NP-hard nature. An effective solution for VRPTW is essential to reduce operational costs, improve customer satisfaction, and improve overall efficiency in logistics operations [5]. VRPTW applications can be found in a variety of industries, including transportation, delivery services, and supply chain management. Advances in computational techniques and algorithms continue to drive progress in completing VRPTW.

Mixed-integer linear programming (MILP) is an optimization method used to determine the best possible solution for an issue by maximizing or minimizing the function of the goal while meeting a set of constraints [6]. MILP combines linear programming (LP) and integer programming (IP), where some of the decision variables are integers and others are real (continuous) numbers [7]. Although MILP can be used to complete VRPTW, it usually requires a long computational time [8].

VRP have many practical implementations, especially for transportation and distribution, which increase efficiency and reduce shipping costs [9]. Yang et al [6] proposed a MILP model for this problem, with the goal of reducing overall fixed and variable costs. The results of this study show that MILP model uncover the best solution and provide a reliable viable solution for small-to-medium businesses.

Dondo et al [10] developed a new model-based improvement methodology for VRPTW problems with multi-depot heterogeneous fleets to improve the initial solution through the solution of a series of MILP mathematical problems that allow inter-tour node exchange and node reordering on each route. Kenaka et al [11] proposed a VRPTW model and combined it with multiple trips (VRPMTTW) to compare the environmental structure applied in local search (LS) for VRPMTTW. Bouyahyiouy et al [12] using the problem of full-load, multi-depot, and window-of-time vehicle routing (SFTMDVRPTW) to develop a solution in which a series of truck routes serve a selected portion of a select number of full-load orders to maximize the total profit earned from those orders. For data processing, the results of the computation were performed on a new dataset containing 30 randomly generated instances of problems ranging from 16 to 30 orders using the CPLEX software. The findings prove that their model has provided a good solution in a reasonable time. Comert et al [13] proposed a new approach to VRP problems by combining them with hard time windows (VRPHTW). Comert et al [13] using this approach to solve the problem of supermarket chains in India. Rave et al [14] proposed a model by solving the problem of determining route location with time windows and load-dependent travel times. They formulated this problem as a mixed integer linear program and introduced adaptive large environment search with problem-specific procedures for micro hub placement and problem-specific operators to solve larger instances.

This study addresses a modified Vehicle Routing Problem with Time Windows (VRPTW) by incorporating constraints such as vehicle capacity, delay times, penalty fees for delays, rigid and flexible time windows, and demand quantities. The model explores whether allowing delays, by introducing flexible time windows, can lead to more optimal routing solutions. Any delays incurred are penalized through a fee structure within the model. There is one original depot for the company that will serve its 15 customers. The company wants to minimize transportation and penalty costs that must be incurred to meet customer demand. Until now, no previous research related to VRPTW has been found that considers rigid and flexible time, penalty costs, vehicle types, and distance.

2. Research Method

2.1. Defining the problem

The initial step in solving route optimization and scheduling problems is to define and describe the problem. The problem that will be solved in this study is to minimize the transportation and penalty costs that will be passed by a company to its customers by considering vehicle capacity, delay times, penalty fees for delays, rigid and flexible time windows, distance, and demand numbers. This model is necessary to determine whether allowing time windows to be delayed within a certain range can result in significantly better routing solutions.

The problem consists of 1 depot and 15 customers. The number of vehicles used is as many as 2 vehicles with the capacity of vehicle 1 smaller than the capacity of vehicle 2. Then, vehicle 1 can only serve customers whose demand are less or equal to 10 kg. Complete data can be seen in Appendix A. The problem will be solved by the exact MILP method and compared with the formulation of VRPTW with flexible time and VRPTW with rigid time.

2.2. Formulating problems

The next stage is to formulate the problem based on the method used, namely mixed-integer linear programming (MILP).

2.3. Running formulations on LINGO software

Pre-designed formulations are tested or run using LINGO software to obtain results. Testing on the LINGO software was carried out on 3 times window conditions, namely rigid, flexible, and rigid and flexible. A rigid time window represents a model where exceeding the time window is not permitted. The flexible time window allows the time window to be exceeded without any time restrictions. Finally, the combined rigid and flexible time window model allows for exceeding the time window, but only within a specified time range.

2.4. Drawing conclusions

Conclusions are made after the results of LINGO are obtained In this conclusion, it is also determined which is the most optimal and feasible solution.

3. Result and Discussions

The observed system in this study is a modified Vehicle Routing Problem with Time Windows (VRPTW), which incorporates constraints such as vehicle capacity, delay times, penalty fees for delays, rigid and flexible time windows, distance, and demand quantities. The model assumes that vehicles must adhere to the specified capacity, with penalties applied for any delays. The process begins by determining the optimal route based on the time window conditions: rigid, flexible, or a combination of both. Following the system definition, a mathematical model is developed to optimize vehicle routes under these conditions.

The notation used to formulate the modified VRPTW problem is described as follows:

$vcost$ = Transportation cost per km
i = Starting point or customer being served
j = Destination or the next customer to be served
k = Represents the index of the vehicle used to serve the route from *i* to *j*
c_{ij} = Distance from point *i* to point *j*
x_{ijk} = A binary variable that indicates whether vehicle k takes a route from location *i* to location *j*
P = Penalty fee per minute
l'_{ik} = Delay time at location *i* for vehicles *k*
d_i = Number of customer requests i (unit)
Q = Vehicle capacity (unit)
V = The set of *i* and *j*
K = The set of *k*
y_{ik} = Arrival time at location *i* for vehicles *k*
s_i = Service time at the destination *i* (minute)
t_{ij} = Travel time from point i to point j (minute)
M = Large constant (big-M)
a_i = The start time *window* time at point *i* (minutes measured from the beginning of distribution process)
b_i = End time *window* at point *i* (minutes measured from the beginning of distribution process)
w = The maximum number of requests that a vehicle can serve (unit)
B = The maximum demand from a customer that can be carried by vehicle 1

The formulation of the modified VRPTW model is shown as follows:

Objective Function

$$\text{Min}= \sum_{k\in K}\sum_{i\in V}\sum_{j\in V} vcost_k.c_{ij}.x_{ijk} + P.\sum_{k\in K}\sum_{i\in V} l'_{ik} \quad (1)$$

Constraints

$$\sum_{k\in K}\sum_{j\in V} x_{ijk} = 1 \ \forall i \in V \setminus \{0\} \quad (2)$$

$$\sum_{j\in V} x_{0jk} = 1 \ \forall k \in K \quad (3)$$

$$\sum_{i\in V} x_{i0k} = 1 \ \forall k \in K \quad (4)$$

$$\sum_{j\in V} x_{ijk} - \sum_{j\in V} x_{jik} = 0 \ \ \forall k \in K, \forall i \in V \setminus \{0\} \quad (5)$$

$$\sum_{i\in V} d_i \sum_{j\in V} x_{ijk} \leq Q \ \ \forall k \in K \quad (6)$$

$$y_{ik} + s_i + t_{ij} - M(1 - x_{ijk}) \leq y_{jk} \ \ \forall k \in K, \forall i \in V, \forall j \in V, i \neq j \quad (7)$$

$$a_i \leq y_{ik} \ \ \forall k \in K, \forall i \in V \quad (8)$$

$$y_{ik} + s_i \leq b_i + l'_{ik} \ \ \forall k \in K, \forall i \in V \quad (9)$$

$$l'_{ik} \leq B \ \forall k \in K, \forall i \in V \quad (10)$$

$$\sum_{i\in V} x_{ij1} .d_j <= w \ \forall j \in V \setminus \{0\} \quad (11)$$

$$x_{ijk} = \{0, 1\} \quad (12)$$

Equation (1) is a function of the goal to minimize transportation costs and penalty costs. Furthermore, equation (2) ensures each customer is visited only once. Equations (3) and (4) show that each vehicle must exit the depot and return to the depot. In addition, equation (5) ensures the consistency of vehicle flow. Equation (6) confirms the limit of vehicle capacity. Further, equation (7) explains the time of arrival and service. Then, equation (8) confirms that the customer must arrive after the time window is opened. Equation (9) explains that the service must be completed before the window closes or is allowed to pass but is subject to a penalty. Meanwhile, equation (10) explains the maximum time of delay. Furthermore, equation (11) confirms that vehicle 1 can only serve a customer with a certain number of requests. Finally, equation (12) explains that the decision variable must have a value of 0 or 1.

The modification of the VRPTW formulation carried out includes equations (1), (10), and (11). Equation (1) shows the function of objectives to minimize the total cost of transportation and penalty costs. Transportation costs are obtained by multiplying the cost of vehicle transportation per kilometer by the distance traveled. The penalty cost is obtained by multiplying the penalty cost per minute by the total delay time. Furthermore, equation (10) limits the maximum delay of delivery only to a certain time. This equation allows the vehicle to go beyond the time window, but the delay of the vehicle is not too long. In addition, equation (11) shows that vehicle 1 can only serve a certain number of customers with a certain number of requests.

Next, the model is tested by solving problems on the Solomon dataset VRPTW R101 customer number 1 - 16. The data contained in the Solomon data is incomplete, so changes are made so that the data set matches the problem to be solved. A comparison of the results for each model using the Lingo software can be seen in Table 1.

Table 1. Comparison of Results for Each Model

Information	Flexible Time Window	Rigid Time Window	Flexible and Rigid Time Windows
Computation Time (Second)	413.26	220.97	530.79
Objective Function Value (Rp)	541,897	561,965	545,142
Number of Delay	1 Customer - 83 minutes	-	1 Customer - 31 minutes

VRPTW models with rigid and flexible time windows have longer compute times than flexible time window models or rigid time window models. However, the value of the objective function and the delay time provided by the rigid and flexible time window model is more reasonable. The rigid and flexible time window model limits the delay time to a maximum of 60 minutes so that the resulting solution does not allow delays of more than 60 minutes. The resulting values of each decision variable for each time window model can be seen in Appendix B. A visualization of the plot of the solution results from the three types of models can be seen in Figure 1 below.

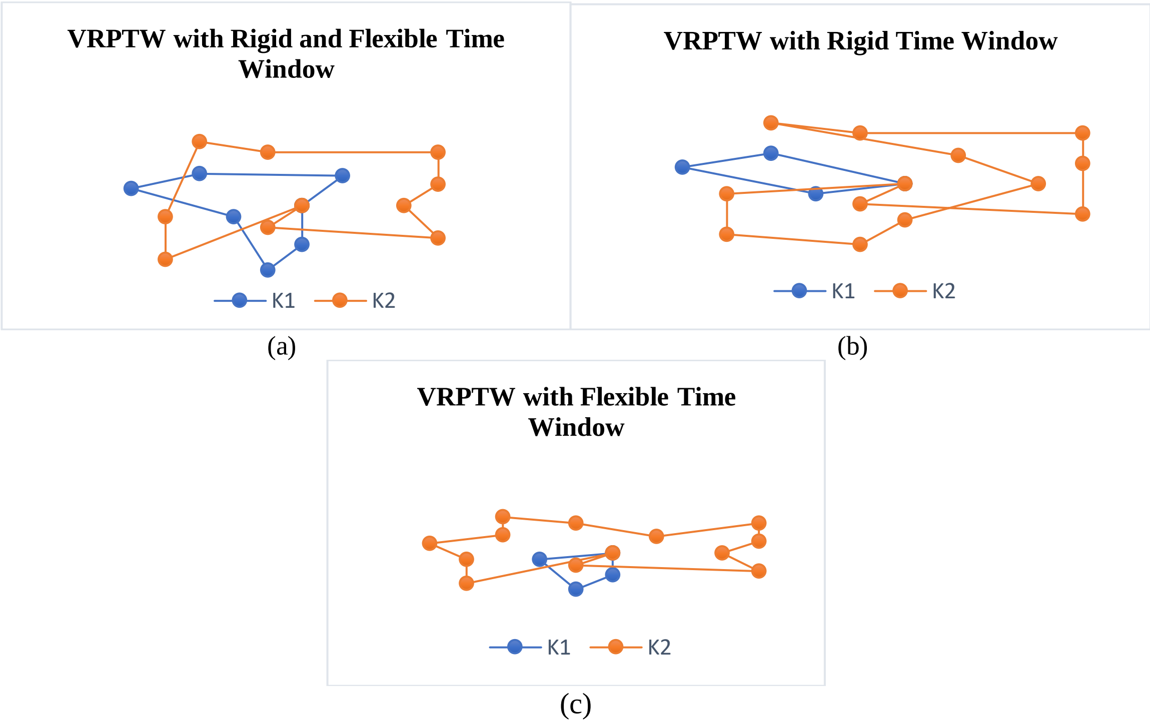

Figure 1. (a) Comparison of VRPTW Solution Results with Rigid and Flexible Time Window; (b) Rigid; and (c) Flexible.

4. Conclusion

VRP problems are common problems faced by companies or business people in serving customers. There are many types of route problems, one of which is VRPTW which is modified with some additional constraints. In this study, the problem of modified VRPTW is solved by considering the constraints of vehicle capacity, delay time, penalty fee for delay, rigid and flexible time window, and number of requests.

In general, this study solves the VRPTW problem of a company by classifying the time window into three groups, namely rigid, flexible, and rigid and flexible. Based on the three types of time window models, the results were obtained: a total cost of Rp 541,897 and 1 customer was served late for 83 minutes for the flexible time window model, the cost was Rp 561,965 and no customer was served late for the rigid time window model, and finally for the rigid and flexible time window model a total cost of Rp 545,142 and 1 customer was served late for 31 minutes. Each model has its own advantages, but the rigid time window model provides a solution that there is no customer served late and the cost difference is not significant or can be said to be very small compared to other time window models.

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Appendix A. Data

A.1. *Dataset*

Customer	Coordinate-X	Coordinate-Y	Demand	Initial Service Time	Final Service Time	Service Time
1	35	35	0	0	828	0
2	41	49	10	161	615.6	10
3	35	17	7	50	216	10
4	55	45	13	116	453.6	10
5	55	20	19	149	572.4	10
6	15	30	26	34	158.4	10
7	25	30	3	99	392.4	10
8	20	50	5	81	327.6	10
9	10	43	9	95	378	10
10	55	60	16	97	385.2	10
11	30	60	16	124	482.4	10
12	20	65	12	67	277.2	10
13	50	35	19	63	262.8	10
14	30	25	23	159	608.4	10
15	15	10	20	32	151.2	10
16	30	5	8	61	255.6	10

A.2. *Dataset (Cont.)*

Data	Value
Vehicle Speed	40 Km/hour
Vehicle 1 Capacity	100 units
Vehicle 2 Capacity	300 units
Vehicle 1 Transportation Cost /Km	Rp 1000
Vehicle 2 Transportation Cost /Km	Rp 2000
Penalty Fee/Minute	Rp 500
Time Allowed to Be Late	60 minutes

Appendix B. Decision Variable Result

B.1. Decision Variable Result

VRPTW					
Flexible and Rigid		Rigid		Flexible	
K1	K2	K1	K2	K1	K2
1	1	1	1	1	1
3	15	7	6	7	15
16	6	9	15	16	6
7	12	8	16	3	9
9	11	1	3	1	8
8	10		13		12
2	4		2		11
1	13		12		2
	5		11		10
	14		10		4
	1		4		13
			5		5
			14		14
			1		1